FXA in Data Mining

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Outline

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Consideraciones Iniciales. . .

En un interesante "final remarks" de su artículo seminal sobre FCA, Wille renunció a cualquier atisbo de exhaustividad en su propuesta de reestructuración de la teoría de retículos y recomendó:

"Besides the interpretation by hierarchies of concepts, other basic interpretations of lattices should be introduced; . . . "[a]

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Maslow acuño una metáfora imprescindible para el científico/ingeniero (matemáticos incluidos)

"... I suppose it is tempting, if the only tool you have is a hammer, to treat everything as if it were a nail. "[a].

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A. Rényi es famoso por sus aforismos (a menudo atribuidos a P. Erdös, su colega y amigo.)

"Un matemático es una máquina de transformar café en teoremas."

Con esta perspectiva, debieramos considerar. . .

- Qué otra información porta un contexto formal.
- Qué pueden ser conceptualizaciones alternativas de esa información.
- ¡Esta es una consideración epistemológica fundamental para no limitar FCA!
- ¿Cómo utilizar productivamente el café que tenemos?

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El Teorema Fundamental del FCA, Extendido (1/3)

Theorem

Sea el contexto formal (G, M, R) en donde,

- *G es un conjunto de objetos formales,*
- *M es un conjunto de attributos formales,*
- *$I \in 2^{g \times m}$ es una relación de incidencia.*

Entonces

(a) *La fase de análisis del contexto.*

  (i) *The polar operators $\cdot^{\uparrow} : 2^G \to 2^M$ and $\cdot^{\downarrow} : 2^M \to 2^G$.*

$$A^{\uparrow} = \{m \in M \mid \forall g \in A, gIm\} \qquad B^{\downarrow} = \{g \in G \mid \forall m \in B, gIm\}$$

form a Galois connection $(\cdot^{\uparrow}, \cdot^{\downarrow}) : 2^G \rightleftarrows 2^G$ whose formal concepts are the pairs (A, B) of closed elements such that $A^{\uparrow} = B \Leftrightarrow A = B^{\downarrow}$ whence

$$\mathfrak{B}(G, M, I) = \{(A, B) \in 2^G \times 2^M \mid A^{\uparrow} = B \Leftrightarrow A = B^{\downarrow}\}$$

El Teorema Fundamental del FCA, Extendido (2/3)

Theorem

(a) *Concepts are partially ordered with the hierarchical order as*

$$(A_1, B_1) \leq (A_2, B_2) \Leftrightarrow A_1 \subseteq A_2 \Leftrightarrow B_1 \supseteq B_2 .$$

and the set of formal concepts with the hierarchical order $\langle \mathfrak{B}(G, M, I), \leq \rangle$ is a complete lattice $\underline{\mathfrak{B}}(G, M, I)$ called the concept lattice of (G, M, I).

(b) *In $\underline{\mathfrak{B}}(G, M, I)$ infima and suprema are given by:*

$$\bigwedge_{t \in T} (A_t, B_t) = \left(\bigcap_{t \in T} A_t, \left(\bigcup_{t \in T} B_t \right)^{\uparrow} \right) \quad \bigvee_{t \in T} (A_t, B_t) = \left(\left(\bigcup_{t \in T} A_t \right)^{\downarrow}, \bigcap_{t \in T} B_t \right)$$

(c) *The mappings $\overline{\gamma} : G \to V$ and $\overline{\mu} : M \to V$*

$$g \mapsto \overline{\gamma}(g) = (\{g\}^{\uparrow\downarrow}, \{g\}^{\uparrow}) \qquad m \mapsto \overline{\mu}(m) = (\{m\}^{\downarrow}, \{m\}^{\downarrow\uparrow})$$

are such that $\overline{\gamma}(G)$ is supremum-dense in $\underline{\mathfrak{B}}(G, M, I)$, $\overline{\mu}(M)$ is

El Teorema Fundamental del FCA, Extendido (3/3)

Theorem

(b) *La fase de síntesis del contexto*

(i) *A complete lattice $\mathscr{V} = \langle V, \leq \rangle$ is isomorphic to (read can be built as) $\underline{\mathfrak{B}}(G, M, I)$ if and only if there are mappings $\overline{\gamma} : G \rightarrow V$ and $\overline{\mu} : M \rightarrow V$ such that*

- *$\overline{\gamma}(G)$ is supremum-dense in $\mathscr{V}$, $\overline{\mu}(M)$ is infimum-dense in $\mathscr{V}$, and*
- *gIm is equivalent to $\overline{\gamma}(g) \leq \overline{\mu}(m)$ for all $g \in G$ and all $m \in M$.*

(ii) *In particular, $\mathscr{V} \cong \underline{\mathfrak{B}}(V, V, \leq)$ using the assignments $G := J(\mathscr{V})$ and $M := M(\mathscr{V})$ where these are the sets of join- and meet-irreducibles, respectively, of $\mathscr{V}$.*

Una Especie de Transformada. . .

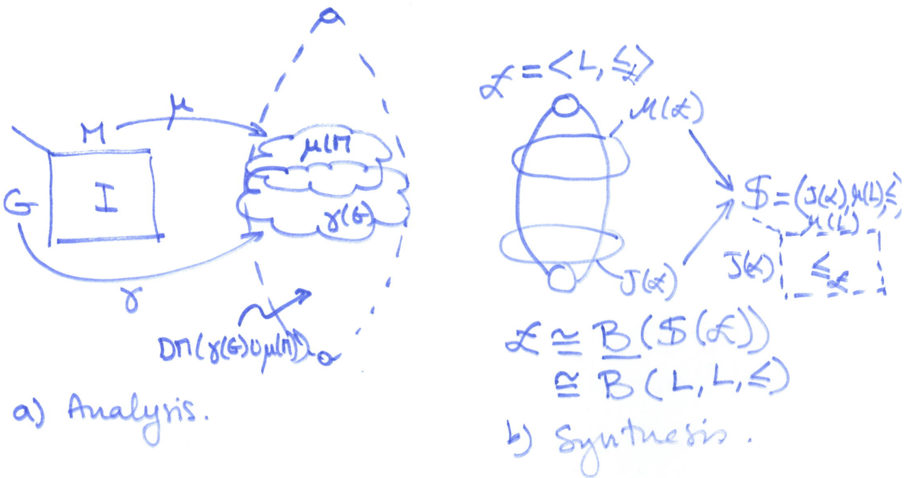

Figure: Las dos fases de FCA como una Transformada de Contextos: a) Análisis y b) Síntesis.

FCA tiene “Puntos Ciegos”

Los polares del contexto generan relaciones de equivalencia

Usando las funciones generadoras de contexto:

$$(g_1, g_2) \in \ker \overline{\gamma} \iff \overline{\gamma}(g_1) = \overline{\gamma}(g_2)$$

$$(m_1, m_2) \in \ker \overline{\mu} \iff \overline{\mu}(g_1) = \overline{\mu}(g_2)$$

El contexto purificado o reducido resintetizado por la parte de síntesis del Teorema Fundamental es

$$\mathbb{S}(\mathfrak{B}(G, M, I)) = (G/\ker \overline{\gamma}, M/\ker \overline{\mu}, I')$$

donde $([g]_{\ker \overline{\gamma}}, [m]_{\ker \overline{\mu}}) \in I' \iff gIm$.

Los elementos individuales de tales clases no están contenidos en $\mathscr{V}$ y no se pueden recuperar.

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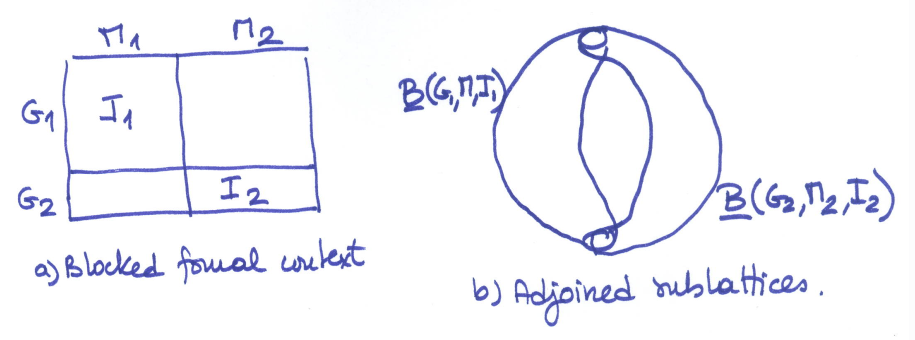
FCA tiene “Espejismos” (I)

Los contextos formales diagonales por bloques son “especiales”.

- Cada bloque es independiente de los demás.
- Lo esperable es que cada bloque genere su propio retículo conceptual.

Pero FCA fuerza a que los retículos compartan $\top$ y $\bot$.



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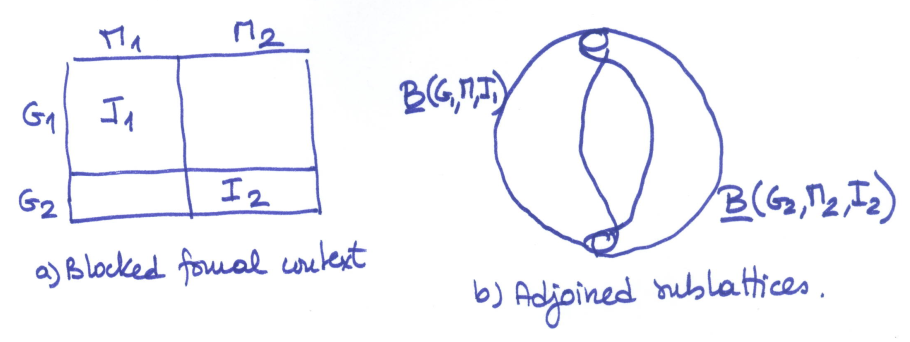

En resumen. . .

FCA no extrae *toda* la información de un contexto formal.

- FCA ignora las equivalencias entre objetos y atributos inducidas por los polares.

FCA introduce cierta "información visual" en el retículo.

- FCA fuerza relaciones jerárquicas donde no las hay.

Necesitamos otras "lentes" para mirar en los "puntos ciegos" y deshacer los "espejismos" del FCA sobre un contexto formal.

Formal XX Analysis

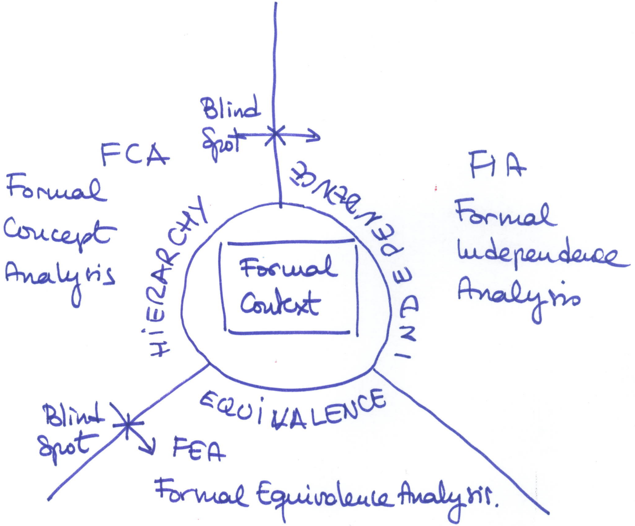

Figure: A variety of formal analysis around a formal context are conceivable: formal concept analysis around the notion of hierarchy as captured by upper

Outline

1. Motivación
2. Formal XXX Analysis
3. Aplicación: Clasificación multietiqueta

Componentes de un "Formal X Analysis"

Si analizamos el Teorema Fundamental, vemos que hay ciertos "ingredientes" fundamentales.

- Una conexión (adjunción) de Galois entre los conjuntos potencia de objetos y atributos, de forma que los pares de subconjuntos estables capturen un "sabor" del retículo.
  - FCA => Los conjuntos de cotas sup. e inf. capturan "jerarquía"
- Un "gránulo" que captura nuclearmente dicho "sabor".
  - FCA => Supremo- e ínfimo-irreducibles.
- Un par de funciones(no inyectivas!) de los objetos formales y los atributos en los gránulos:
  - FCA => $\overline{\gamma}$ y $\overline{\mu}$ generadores de conceptos.
- Un teorema de representación en función de los "gránulos" de determinada capacidad(universal, lo mejor).
  - FCA => Teorema de Dedekind-McNeille $\mathscr{L} \equiv DM(\mathscr{J}(\mathscr{L}) \cup \mathscr{M}(\mathscr{L}))$

Ejemplo: Formal Independence Analysis

Sea un orden parcial finito $\mathscr{P} = \langle P, \leq \rangle$

- Las anticadenas capturan la noción de no-relación.
- No conocemos un teorema de representación pero existe...

Theorem (Behrendt's representation theorem)

Let $\mathscr{L} = \langle L, \leq \rangle$ be a lattice. Then there exists a poset $\mathscr{P} = \langle P, \leq_P \rangle$ such that $|P| = 2|L|$, where any chain has at most 2 elements and such that $\mathscr{L}$ is isomorphic to the lattice of maximal antichains of $(P, \leq_P)$.

$$\mathscr{L} \cong MA(\mathscr{P})$$

- Esto sugiere que las anticadenas maximales (y minimales para el dual) son los “gránulos”.
- Existen conexiones de Galois entre los conjuntos de cotas superiores e inferiores y las anticadenas maximales y minimales.

El Teorema Fundamental del FIA (1/2)

Theorem (Basic theorem of formal independence analysis (Valverde, Peláez, Cabrera, Cordero, Ojeda, 2018))

(a) *The context analysis phase: Given a formal context (G,M,I),*

(i) *The context operators $\cdot^{\sim} : 2^G \to 2^M$ and $\cdot_{\sim} : 2^M \to 2^G$*

$$\alpha^{\sim} = M \setminus \uparrow \alpha \qquad \beta_{\sim} = G \setminus \downarrow \beta$$

form a right-Galois connection $(\cdot^{\sim}, \cdot_{\sim}) : (2^G, \subseteq) \rightharpoonup (2^M, \subseteq)$ whose formal tomoi are the pairs (α, β) such that $\alpha^{\sim} = \beta$ and $\alpha = \beta_{\sim}$.

(ii) *The set of formal tomoi $\mathfrak{A}(G,M,I)$ with the relation*

$$(\alpha_1, \beta_1) \leq (\alpha_2, \beta_2) \text{ iff } \alpha_1 \supseteq \alpha_2 \text{ iff } \beta_1 \subseteq \beta_2$$

is a complete lattice, which is called the tomoi lattice of (G,M,I) and denoted $\underline{\mathfrak{A}}(G,M,I)$, where infima and suprema are given by:

$$\bigwedge_{t\in T} (\alpha_t, \beta_t) = \left(\bigcup_{t\in T} \alpha_t, \left(\bigcap_{t\in T} \beta_t \right)_{\sim}^{\;\sim} \right) \quad \bigvee_{t\in T} (\alpha_t, \beta_t) = \left(\left(\bigcap_{t\in T} \alpha_t \right)^{\sim}_{\;\sim}, \bigcup_{t\in T} \beta_t \right)$$

El Teorema Fundamental del FIA (2/2)

Theorem

(a) *The mappings $\overline{\gamma}: G \to \underline{\mathfrak{A}}(G,M,I)$ and $\overline{\mu}: M \to \underline{\mathfrak{A}}(G,M,I)$*

$$g \mapsto \overline{\gamma}(g) = (\{g\}^{\sim}{}_{\sim}, \{g\}^{\sim}) \qquad m \mapsto \overline{\mu}(m) = (\{m\}_{\sim}, \{m\}_{\sim}{}^{\sim})$$

are such that $\overline{\gamma}(G)$ is infimum-dense in $\underline{\mathfrak{A}}(G,M,I)$, $\overline{\mu}(M)$ is supremum-dense in $\underline{\mathfrak{A}}(G,M,I)$.

(b) *The context synthesis phase: Given a complete lattice $\mathbb{L} = \langle L, \leq \rangle$*

(i) *$\mathbb{L}$ is isomorphic to[a] $\underline{\mathfrak{A}}(G,M,I)$ if and only if there are mappings $\overline{\gamma}: G \to L$ and $\overline{\mu}: M \to L$ such that*

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(ii) *In particular, $\mathbb{L} \cong \underline{\mathfrak{A}}(L, L, \not\geq)$ and, if L is finite, $\mathbb{L} \cong \underline{\mathfrak{A}}(M(\mathbb{L}), J(\mathbb{L}), \not\geq)$ where $M(\mathbb{L})$ and $J(\mathbb{L})$ are the sets of meet- and join-irreducibles, respectively, of $\mathbb{L}$.*

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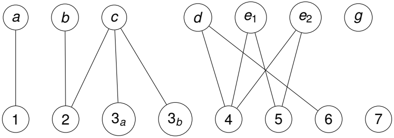
[a] Read *can be built as*.

Example context

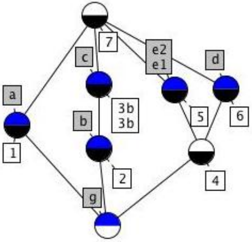
$\mathbb{K}_1$	a	b	c	d	e1	e2	g
1	×						
2		×	×				
3a			×				
3b			×				
4				×	×	×	
5					×	×	
6				×			
7							

(a) Tabular representation of $\mathbb{K}_1$

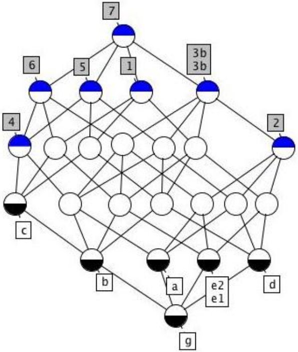


(b) Bipartite graph representation of $\mathbb{K}_1$

Figure: **Equivalent representations of an example context $\mathbb{K}_1 = (G, M, I)$.** (a)) tabular representation. (b)) bipartite graph representation.



(a) Concept lattice $\mathfrak{B}(G, M, I)$



(b) Tomoi lattice $\underline{\mathfrak{A}}(G, M, I)$

Figure: **Two different lattices for context** $\mathbb{K}_1 = (G, M, I)$ (a) lattice of formal concepts *showing* three adjoint sublattices, and (b) lattice of formal tomoi, *describing* the three adjoint sublattices.

Carrying formal analysis on partitions

We have seen that:

- There are partitions $\ker \overline{\gamma}$ and $\ker \overline{\mu}$ related to a formal context.
- The FCA procedure is blind to the classes in these partitions, that is, there is an information loss in the process of FCA.
- There is a notion of “mutual determination” of objects and attributes, whether from the qualitative point of view of Psychology and Cognition, or from the purely quantitative of Information Theory.

Define the *coarsening order*:

Let $\mathrm{Part}(G)$ denote the set of partitions over G.

$$\pi \leq \sigma \iff \forall g_1, g_2 \in G, g_1 \equiv g_2(\pi) \text{ implies that } g_1 \equiv g_2(\sigma)$$

The lattice of partitions of a set

Theorem

Let G be a set. Then $\mathrm{Part}(G) = \langle \Pi(G), \subseteq \rangle$ is a complete lattice, called the partition lattice (or equivalence lattice of G) where:

- *The bottom of $\mathrm{Part}(G)$ is $\iota_G = \{\{g\} \mid g \in G\}$ is the set of trivial blocks.*
- *The top of $\mathrm{Part}(G)$ is $\omega_G = \{G\}$.*
- *The meet of partitions $\{\pi_i \mid i \in I\}$ is defined, for all $g_1, g_2 \in G$ as:*

$$g_1 \equiv g_2(\wedge_{i \in I} \pi_i) \iff \forall i \in I, g_1 \equiv g_2(\pi_i)$$

- *The join of partitions $\{\pi_i \mid i \in I\}$ is defined, for all $g_1, g_2 \in G$ as:*

$$g \equiv d(\vee_{i \in I} \pi_i) \iff$$

there is a natural number n, a subset $J = \{i_0, \ldots, i_n\} \subseteq I$, and $g_0, \ldots, g_{n+1} \in G$ such that $g = g_0, \ldots, d = g_{n+1}$ and $g_j = g_{j+1}(\pi_{i_j})$, for $0 \leq j \leq n$

More partitions

Theorem (Continued)

- *The atoms of* $\mathrm{Part}(G)$ *are the partitions with exactly one non-trivial block and this block has two elements.*
- *The co-atoms of* $\mathrm{Part}(G)$ *are the partitions with exactly two blocks.*
- *The covering relation in* $\mathrm{Part}(G)$ *holds* $\pi \prec \sigma$ *iff* σ *is the result of replacing two blocks of* π *by their union.*

Proposition

Let G be a set and consider $\pi \in \Pi(G)$*. Then,*

- $\uparrow \pi \subseteq \mathrm{Part}(G)$ *is isomorphic to the partition lattice of the set* π*,* $\uparrow \pi \cong \mathrm{Part}(\pi)$*.*
- $\downarrow \pi \subseteq \mathrm{Part}(G)$ *is isomorphic to direct product of* $\mathrm{Part}(X)$ *where* X *ranges over the non-trivial blocks of* π*,* $\downarrow \pi \cong \bigtimes_{X \text{ non trivial in } \pi} \mathrm{Part}(X)$*.*

La “bombilla” de las particiones

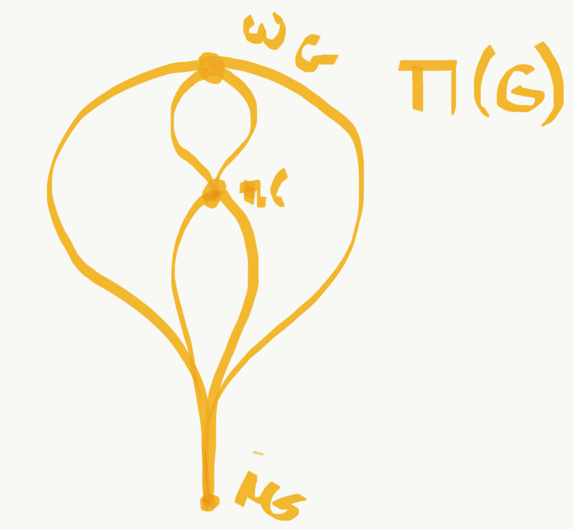


Figure: Sketch of the lattice of partitions with the filter and ideal of $\pi \in \mathrm{Part}(G)$ drawn.

El Teorema Fundamental del FEA (1/2)

Theorem (Basic theorem of formal equivalence analysis (Valverde, Peláez, Cordero, Ojeda, 2019))

(a) *The context analysis phase: Given a formal context* (G, M, I)*,*

(i) *The operators form a left adjunction whose formal equivalences or partitions are the pairs* (π, σ) *such that* $\pi^{\exists}_{\sqcap} = \sigma \iff \sigma^{\forall}_{\sqcap} = \pi$

(ii) *The set of formal partitions* $\mathfrak{P}(\mathbb{K})$ *with the relation* $(\pi_1, \sigma_1) \leq (\pi_2, \sigma_2)$ *iff* $\pi_1 \leq \pi_2$ *iff* $\sigma_1 \leq \sigma_2$ *is a complete lattice, which is called the partition lattice of* $\mathbb{K}$ *and denoted* $\underline{\mathfrak{P}}(\mathbb{K})$*, with infima and suprema given by:*

$$\bigwedge_{t \in T} (\pi_t, \sigma_t) = \left(\bigwedge_{t \in T} \pi_t, \left[\left(\bigwedge_{t \in T} \sigma_t \right)^{\forall}_{\sqcap} \right]^{\exists}_{\sqcap} \right)$$

$$\bigvee_{t \in T} (\pi_t, \sigma_t) = \left(\left[\left(\bigvee_{t \in T} \pi_t \right)^{\exists}_{\sqcap} \right]^{\forall}_{\sqcap}, \bigvee_{t \in T} \sigma_t \right)$$

El Teorema Fundamental del FEA (2/2)

Theorem

(a) *The mappings in* (**??**) $\overline{\gamma}_\Pi(\cdot) : G \to \mathfrak{P}(\mathbb{K})$ *and* $\overline{\mu}_\Pi(\cdot) : M \to \mathfrak{P}(\mathbb{K})$ *are such that* $\overline{\gamma}_\Pi(G)$ *is* $\vee$*-dense in* $\mathfrak{P}(\mathbb{K})$, $\overline{\mu}_\Pi(M)$ *is* $\wedge$*-dense in* $\mathfrak{P}(\mathbb{K})$.

(b) *The context synthesis phase: Given a complete lattice* $\mathbb{L} = \langle L, \leq \rangle$ *Given a complete lattice* $\mathbb{L} = \langle L, \leq \rangle$

(i) $\mathbb{L}$ *is isomorphic to*[a] $\mathfrak{P}(G, M, I)$ *if and only if there are mappings* $\overline{\gamma} : G \to L$ *and* $\overline{\mu} : M \to L$ *such that*

- $\overline{\gamma}(G)$ *is* $\vee$*-dense in* $\mathbb{L}$, $\overline{\mu}(M)$ *is* $\wedge$*-dense in* $\mathbb{L}$, *and*
- $g\, I\, m$ *is equivalent to* $\overline{\gamma}(g) \not\geq \overline{\mu}(m)$ *for all* $g \in G$ *and all* $m \in M$.

(ii) *In particular,* $\mathbb{L} \cong \mathfrak{P}(L, L, \not\geq)$ *and, if* L *is finite,* $\mathbb{L} \cong \mathfrak{P}(J(\mathbb{L}), M(\mathbb{L}), \not\geq)$ *where* $M(\mathbb{L})$ *and* $J(\mathbb{L})$ *are the sets of meet- and join-irreducibles, respectively, of* $\mathbb{L}$.

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[a] Read *can be built as*.

Example context

$\mathbb{K}_0$	a	b	c	d
1	×	×	×	×
2			×	×
3			×	×
4	×	×		
5	×	×		
6	×	×		

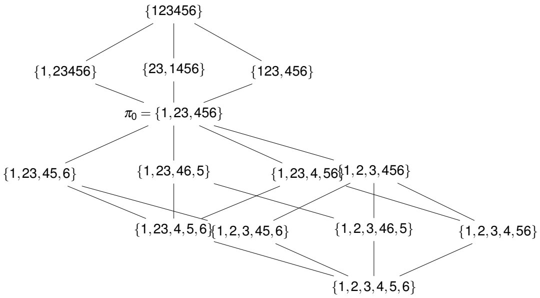
(a) Context $\mathbb{K}_0$

$\mathbb{K}_0^0$	a	c
1	×	×
2		×
4	×	

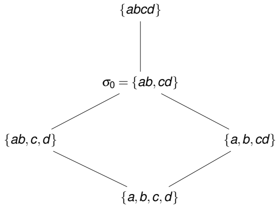
(b) Context $\mathbb{K}_0^0$

Figure: Context $\mathbb{K}_0$, its clarified context $\mathbb{K}_0^0$

The ambient partition lattices



(a) Neighbourhood of $\pi_0 = \{1, 23, 456\}$.



(b) Neighbourhood of $\sigma_0 = \{ab, cd\}$.

The FEA lattice

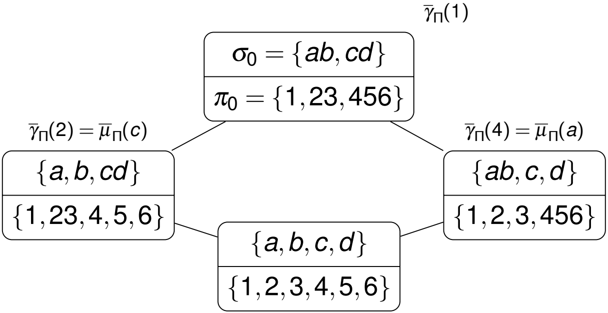


Figure: Partition lattice $\mathfrak{P}(\mathbb{K}_0)$.

Outline

Multi-Label Classification (MLC)

- Let $K \neq \varnothing$ be a set of *labels* and $\overline{K} = 2^L$ be a space of *labelsets*.
- Let $\overline{X} = \mathbb{R}^{mx}$ be a space of *observations*.
- Let $\{(\vec{k}^i, \vec{x}^i)\}_{i=1}^N$ be a collection of labelsets $\vec{k}^i$ assigned to observation vectors $\vec{x}^i$, that we call the *task dataset.*

The Multi-Label Classification task

- Divide the task datasets in two non-overlapping subsets
  - The *training dataset*, $\{(\vec{k}_T^i, \vec{x}_T^i)\}_{i=1}^{N_T}$
  - The *test dataset*, $\{(\vec{k}_E^i, \vec{x}_E^i)\}_{i=1}^{N_E}$

  So $N = N_T + N_E$
- Use the training dataset to build a model: $\hat{h} : \mathbb{R}^{mx} \to 2^L$
- Predict the outputs $\hat{k}_E^i = \hat{h}(\vec{x}_E^i)$ on the testing observation vectors.
- Measure the quality of approximating $\vec{k}_E^i$ by $\hat{k}_E^i$

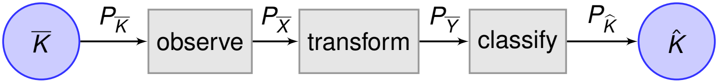
Some datasets...

Name	$N_E + N_T$	M	L	distinct
emotions	391+202	72	6	27
scene	1 211+1 196	294	6	15
yeast	1 500+917	103	14	198

Figure: A selection of multi-label classificaton databases (data from [**tso:kat:vla:10**]). “distinct” refers to distinct label sets ocurring in the training decision data.

Multi-label Classification as an Information Channel

- Supervised Classification as an Information Channel



- It is possible to *assess* this system by entropic means. We have
  - modelling tools, the positive (dual pairs of) semifields (cfr. keynote from ESCIM'17, Entropy'19)
  - "prescription" tools, the balance equations and,
  - data exploratory tools, the entropy triangles and diagrams (PRL'11, ESWA'17,Entropy'18).
- Each tool provides different insight into the technologies implementing each subsystem.
- We want to add FXA tools to these!

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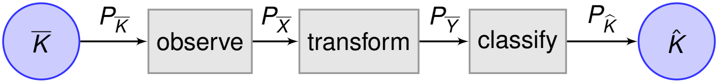

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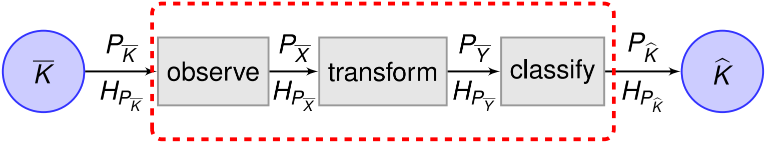
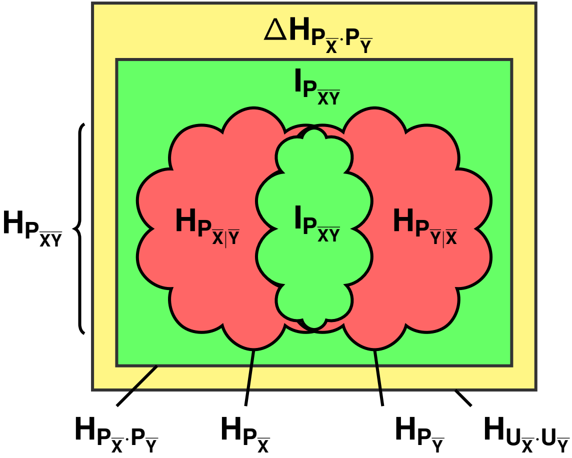
Sidetrack: End-to-End Evaluation of $P_{K\widehat{K}}$

- We consider classification as the transmission of $\overline{K}$.
- For end-to-end evaluation we ignore all details within the outer hatched block:



- The whole chain is considered here as a multiple-input multiple-output (MIMO) block with joint distribution $(\overline{K}, \hat{K}) \sim P_{\overline{K}\hat{K}}$

Our method is based in an exhaustive analysis of the information in $P_{\overline{XY}}$.



Context Modelling of an MLC task

We concentrate on the labelling part

- Call G the set of indices on observation pairs. We keep L as the name of the attribute set.
- Call $\mathbb{K} = (G, L, I)$ where $gIm \iff$ "observation g has label m"

Define training and testing parts, where G_T the index set on the training and G_E the index set on the test observation pairs.

- Call $\mathbb{K}_T = (G_T, L, I \cap G_T \times L)$ and Similarly define $\mathbb{K}_E = (G_E, L, \cap G_E \times L)$
- Clearly $G = G_T \cup G_E$, and for good statistic practice we need $G_T \cap G_E = \varnothing$ so

$$\mathbb{K} \cong \mathbb{K}_T / \mathbb{K}_E \quad (1)$$

  This is the subposition of contexts.
- In this situation, FIA is indicated for task simplification.

MLC Stratified N-fold validation is “solved” by FXA

Statistical validation of models

- This is a process of assessing the quality of statistical models issued from data.
- N-fold validation is quite used:
  - Divide *all* data in *n* parts or *folds*
  - Use $n-1$ folds as the training part and 1 fold as the test part.
  - Permute to carry out a total of *n* models and obtain assessment figures.
  - Assess the distribution of assessment figures.
- *Stratified* n-fold validation is very problematic for MLC

FXA can provide leverage to solve this:

- The Train-Test duality supposes (for instance) that $\mathfrak{B}\mathbb{K}_T \cong \mathfrak{B}\mathbb{K}_E$
- The solution is to carry out stratification modulo $\ker \tilde{\gamma}(\mathbb{K})$

Conclusiones

- Some MLC concepts can be naturally captured by FXA abstractions.
- It would seem that FXA can guide improve the modelling in MLC label space.
- To do likewise with observation space, we would need *numeric* FXA.

¡Gracias!
¿Alguna pregunta?
¿Alguna sugerencia?

CLA 2020

- The 15th International Conference on Concept Lattices and Their Applications
- Tallinn, Estonia
- Fechas: June 29–July 1, 202
- Abstract deadline: January 26, 2020
- Paper deadline: February 2, 2020

¡Enviadnos contribuciones!